1. VERY BASIC NIM

The players are presented with 2 piles of stones. Each move involves taking 1 or more stones from one of the piles. The player who takes the last stone wins.

**Strategy:** Keep the piles balanced (i.e. the same size).

2. STANDARD NIM

The players are presented with an arbitrary number of piles of stones. Each move involves taking 1 or more stones from one of the piles. The player who takes the last stone wins.

**Strategy:** Write the size of each pile in binary. Add the sizes up without carry (XOR also known as nim-sum). The winning move is to always make this sum 0.

3. INDEX-k NIM

The players are presented with an arbitrary number of piles of stones. Each move involves taking 1 or more stones from up to \( k \) piles. The player who takes the last stone wins.

**Strategy:** Write the size of each pile in binary. Add the sizes modulo \( k + 1 \) in each digit without carry. The winning move is to always make this sum 0.

4. GREEDY NIM

The players are presented with an arbitrary number of piles of stones. Each move involves taking 1 or more stones from the largest pile (or one of the largest piles if tied). The player who takes the last stone wins.

**Strategy:** Keep the number of piles tied for largest even.

5. POINTED NIM

The players are presented with an arbitrary number of piles of stones, and a pointer to one pile. Each move involves taking 1 or more stones from the pile pointed to, and then moving the pointer to a different non-empty pile. The player who takes the last stone wins.

**Strategy:** Keep the number of non-empty piles even, and always move the pointer to the smallest pile.

6. BUILDING NIM

The players are presented with an arbitrary number of empty piles, and a storage with some number of stones. This is a 2 stage game.

**Stage 1:** Place 1 or more stones from storage into a pile.

Stage 1 continues until the storage is empty. Next player to move begins stage 2.

**Stage 2:** Play Standard Nim on the resulting board.

**Strategy:** Not fully solved!!

Let there be \( n \) stones and \( \ell \) piles

If \( n \) is even, Player 2 wins.

If both \( n \) and \( \ell \) are odd, Player 2 wins.

If \( \ell = 3 \), then Player 2 wins iff \( n = 2^k - 2 \) for some \( k \).

If \( \ell > 3 \) and \( n \leq \ell + 3 \), Player 2 wins.

If \( \ell = 5 \), the Player 1 wins iff \( n \geq 10 \).